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DEPTHS OF OVERLAND FLOW

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DEPTHS OF OVERLAND FLOW

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ABSTRACT

A discussion of uniform laminar flow is followed by a description and analysis of flow measurements made under several conditions of bed roughness and disturbances by raindrop impact and vegetative cover. A simple modification of the laminar flow equation by inclusion of the ratio of actual to theoretical depth is suggested to adequately represent disturbed viscous flow. Values of the ratio are given for the several conditions that were tested. The conditions included a considerable range in vegetative cover on relatively smooth land surfaces.

INTRODUCTION

Preamble

Information in regard to the nature of overland flow during periods of surface runoff is desirable in the estimation of the effects of land slope and land use change on the volume and rate of surface runoff. A better knowledge of the manner of sheet flow should also aid in the interpretation of soil erosion phenomena and therefore in the evaluation of land treatments used for the control of erosion.

Both of the terms "sheet flow" and "overland flow" seem to have different meanings for different people and are used herein largely because of precedent in the literature. These studies deal, more precisely, with small, unit-width flows. A list of the symbols used, along with their definitions, is included for ready reference.

Laminar Flow

Low sheet flows at very small depths have been shown by Horton, Leach, and Van Vliet (*1*)[1] to be laminar or viscous. According to them the velocity within the flowing water varies parabolically with distance above the bed, being zero at the bed and a maximum at the water surface.

Their complete expression for uniform flow is

$$v = \frac{gS}{\nu}\left(Dd - \frac{d^2}{2}\right) \tag{1}$$

where:
- v = velocity at distance d above the bed,
- g = acceleration due to gravity
- S = slope of the bed and water surface,
- ν = kinematic viscosity = coefficient of viscosity divided by fluid density, and
- D = depth of flow.

They also determined by integration of equation (1) that the average velocity is

$$V = \frac{gSD^2}{3\nu} \tag{2}$$

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[1] Italic numbers in parentheses refer to Literature Cited at end of report.

that the surface velocity is 3/2 the average velocity; and that the flow per unit width in a wide shallow channel is

$$q = \frac{gSD^3}{3\nu} \tag{3}$$

It is appropriate here to compare laminar flow formulae for sheet flow and flow in circular tubes. Hagen and Poiseuille are credited with the relation

$$V = \frac{gSr^2}{8\nu} \tag{4}$$

where r is the radius of the tube and S is the rate of energy loss per unit weight of liquid in the direction of flow. Using the hydraulic radius R to represent length, there is for comparison

from (2) $$V = \frac{gSR^2}{3\nu} \quad \text{(for sheet flow and flow between very wide parallel plates)} \tag{5}$$

from (4) $$V = \frac{gSR^2}{2\nu} \quad \text{(for flow in circular tubes)} \tag{6}$$

This indicates that, in the case of laminar flow, the hydraulic radius is not a particularly good representation of effective length. The value of the coefficient in the denominator varies with shape of conduit between 2 and 3.

The Upper Limit of Laminar Flow

It was pointed out by Horton et al. in a very good discussion of the upper limit of laminar flow that the Reynolds number, using the depth as the length factor, is equal to

$$N = \frac{VD}{\nu} = \frac{q}{\nu} \tag{7}$$

and that Jeffreys (2) used this as a criterion. Jeffreys gave a value of N = 310 for the point of change between pure laminar and turbulent flow whereas the data of Horton et al. indicated a value between 548 and 773. At ordinary temperatures these values correspond to flows per foot width of about 0.003 to 0.007 cfs.

An apparently logical criterion, developed by equating laminar and turbulent flow formulae, was given by Horton. However, the largely empirical Manning formula, derived from flow data considerably outside of the range of application, was used to express the turbulent flow relationships. The tacit assumption is also made that with increasing discharge the flow changes abruptly from completely laminar to completely turbulent.

According to the Horton criterion, the velocity below which laminar sheet flow must prevail is

$$V = \frac{\nu}{5n^2 D^{2/3}}$$

$$N = \frac{D^{1/3}}{5n^2} \qquad (8)$$

When the same scheme of equating laminar and turbulent flow formulae is used with some of the other turbulent flow representations, there results for the Reynolds number, below which laminar flow must prevail,

$$N = \frac{3C^2}{g} = \frac{24}{f} = 3\left(\frac{\bar{u}}{\bar{u}*}\right)^2 \qquad (9)$$

where n = the retardance factor in the Manning formula, $V = 1.486R^{2/3}S^{1/2}/n$,
C = coefficient in Chezy's formula, $V = C(RS)^{1/2}$,
f = resistance factor in the Weisbach formula, $S = (f/4R)\ (V^2/2g)$,
$\bar{u}/\bar{u}*$ = Keulegan's notation for the ratio of average velocity to average boundary shear velocity.

Equations (8) and (9) are quite meaningless in the absence of a knowledge of the values of the parameters in the flow range immediately greater than laminar.

Dr. Keulegan (4) demonstrates that his "mean flow ratio" $\bar{u}/\bar{u}*$ is defined in three different ways for turbulent flow, depending upon the hydraulic characteristics of the channel; and in all derivations the laminar boundary layer was ignored, a fact that may be of significance in this application. Using his formula for a hydrodynamically smooth channel, namely

$$\bar{u}/\bar{u}* = V/(gDS)^{1/2} = 3.0 + 5.75 \log D\ (gDS)^{1/2}/\nu \qquad (10)$$

a critical Reynolds number of 427 for the change from turbulent to laminar flow is obtained.

In tests with water and kerosene in a smooth, narrow rectangular channel Straub (6) demonstrates the manner of influence of the kinematic viscosity on the flow. He concludes that the Reynolds critical value for open channels, using the hydraulic radius as the characteristic length, seems to be at about 640, possibly lower.

There is a considerable variation in these estimates, and each is undoubtedly correct for the test condition under which it was derived. It will be shown that other factors besides the bulk Reynolds number, VD/ν, apparently influence the upper limit of viscous type flow, and that the change is from laminar type to an intermediate type rather than to strictly turbulent flow. The question is of interest in this study because the point of change is well within the range encountered in so-called overland flow.

SHEET FLOW MEASUREMENTS

Flow Over a Smooth Bed

A few tests were made at Auburn, Ala., in a tilting channel 2 feet wide by 8 feet long. Figures 1 and 2, pages 4 and 5, show measured depths in comparison with the depths for laminar flow as calculated with the use of equation (3), page 2. Each point generally represents the mean of seven water surface measurements taken equidistant across the channel and 4 feet downstream from the entrance. There was a hard, smooth bed of troweled mortar for these tests. It may be seen that the measured depths agree quite closely with the calculated depths for flows having Reynolds numbers less than about 500 to 700. As the discharge per unit width is increased, the measured depths of flow become increasingly greater than the calculated depths for laminar flow.

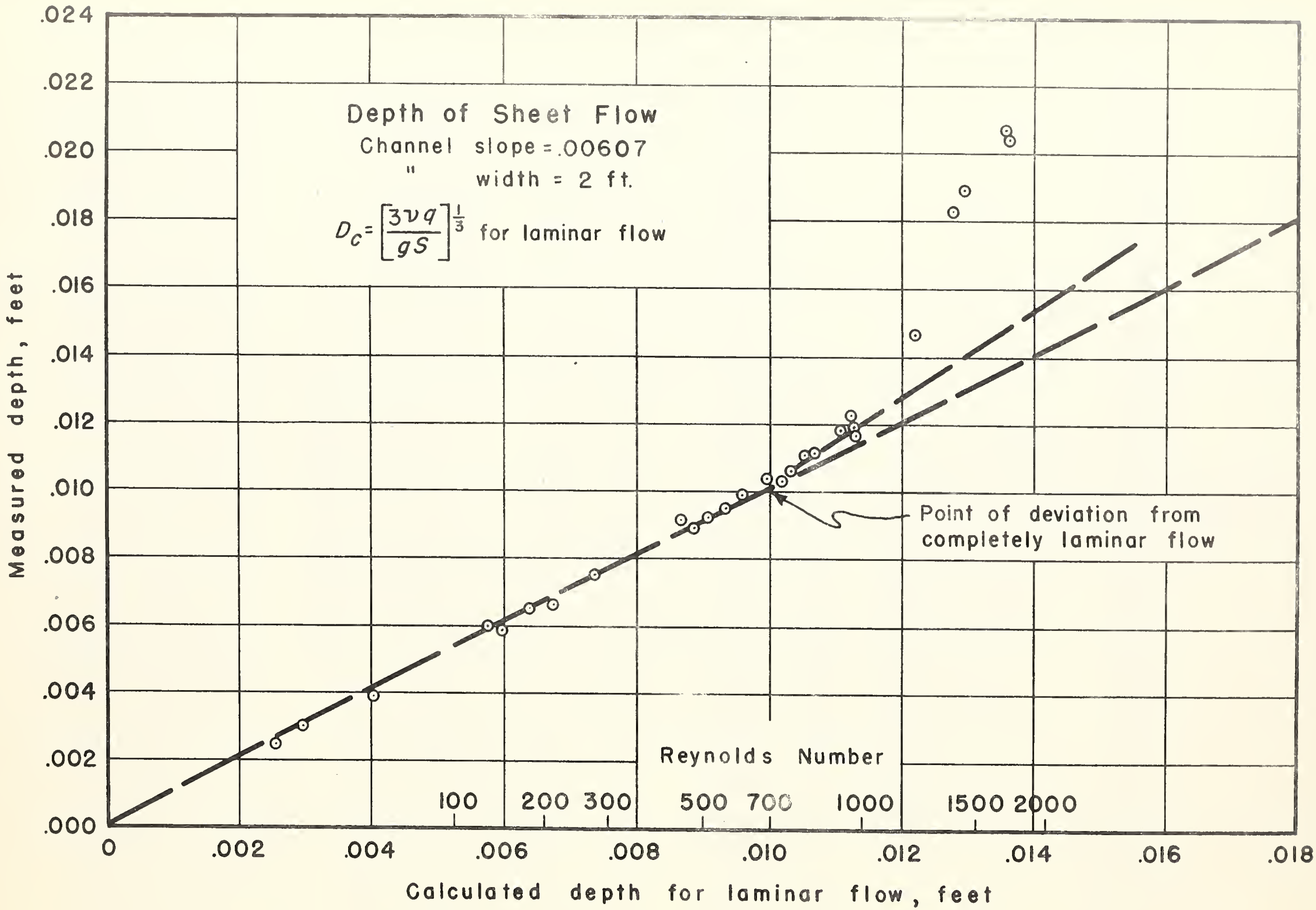

Figure 1. — Depth of sheet flow on a smooth bed (slope = 0.00607)

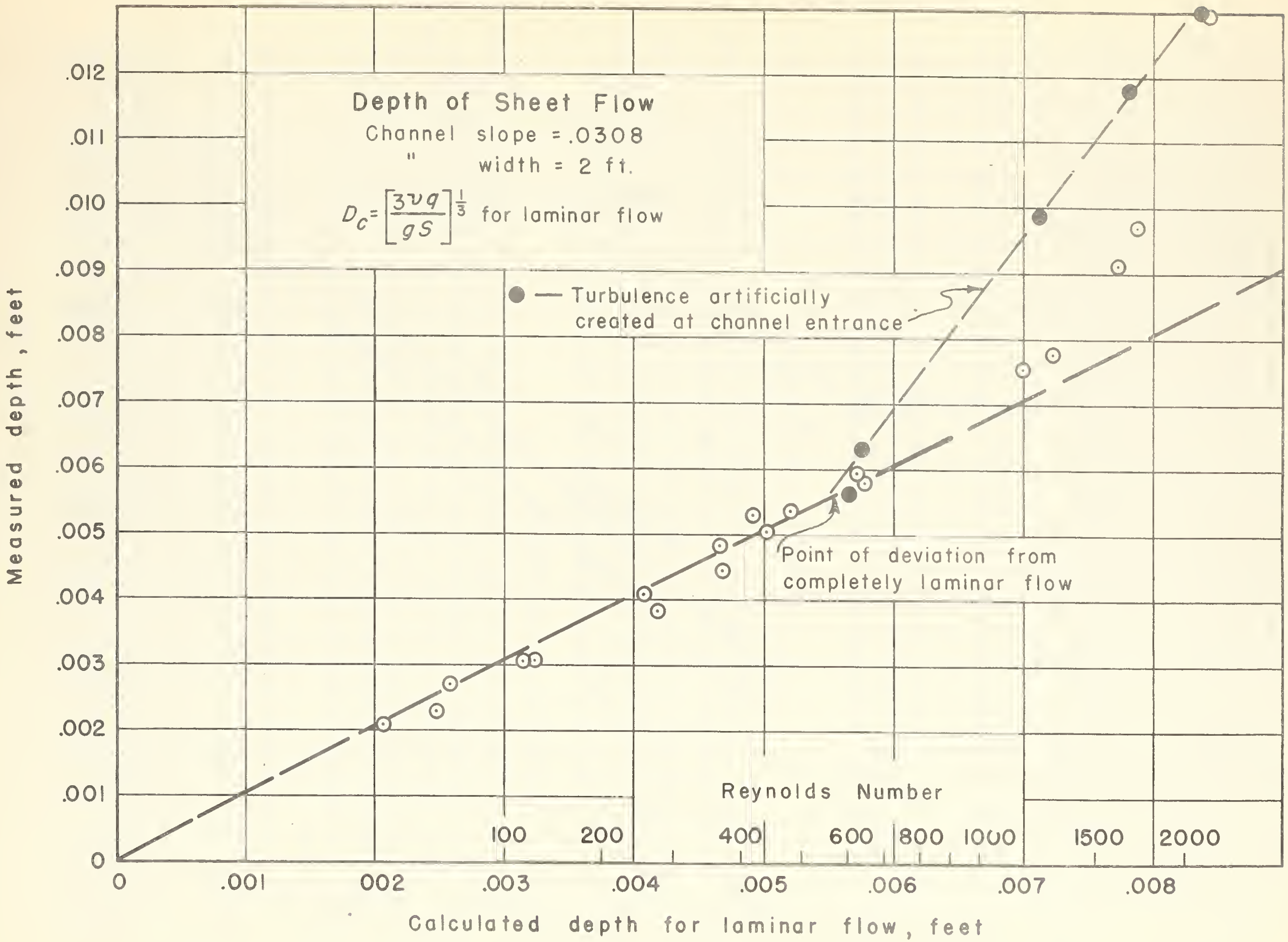

Figure 2.—Depth of sheet flow on a smooth bed (slope = 0.0308)

In the flow range just above the critical the flow is mixed. Turbulent patches form in an apparently fortuitous manner as to position of development, and travel downslope. The frequency of formation of these patches and the channel area covered increases with increasing discharge. The impression was gained that if the channel had been very long, the whole area would have become turbulent at flows above the critical, because, once formed, the turbulent areas seemed to maintain themselves.

With this in mind, a few measurements were made with weak turbulence artificially created at the channel entrance by causing the flow to pass through a grid. The measured depths were considerably greater than had been previously obtained within the Reynolds number range of about 600 to 2,000. Outside of this range there was little, if any, effect. These measurements are represented by solid symbols in figure 2, page 5.

Figure 3, page 7, shows the variation in Manning's n through the range of flows from definitely laminar to definitely turbulent. The value of n in the turbulent flow range was approximately 0.010.

The bed of the channel was purposefully made smooth; but one might conclude that it was hydrodynamically rough, as is indicated by the plotting of the data in figure 4, page 8. Here, Keulegan's mean flow ratio, $\bar{u}/\bar{u}^* = C/g^{1/2}$, is plotted on semilogarithmic paper against the depth of flow. The equation of the straight line, drawn to represent the data in the definitely turbulent flow range, is

$$V/(gDS)^{1/2} = 23.3 + 5.75 \log D \qquad (11)$$

When equation (11) is compared with the logarithmic formula for rough channels, based on Nikuradse's tests, it is determined that the roughness was equivalent to that of sand grains having a diameter of 0.00098 foot = 0.03 cm. This value for the roughness seems a little large.

It is believed to be noteworthy that the mean flow ratio and Chezy's C were constant for the tests with artificially induced turbulence, within the transition range between pure laminar and completely turbulent flow. Flows occurring under natural conditions are often, if not usually, disturbed. The value of $V/(gDS)^{1/2}$ within this range was 12.8. Since slope and bed roughness were not varied, this value is known to be applicable only to the particular test conditions.

Depth of Viscous Flow Over a Rough, Pitted Bed

Similar flow measurements, but with a rough, pitted bed, were also made in the 8-foot channel. The channel bed was prepared by sprinkling to incipient runoff a straight-edged mixture of dry sand and Portland cement. The resultant surface was quite uniformly rough and pitted by spray-drop impact. Measurements at 0.01-foot intervals across the 2-foot wide channel gave a mean bed elevation of 0.0090 foot above an arbitrary datum. The average deviation in elevation from a moving average of 10 readings (0.1 ft.) was $\pm$ 0.00132 foot. The extremes were about $\pm$ 0.0045 foot. Probably because of the pitted nature of the surface, there appeared to be a tendency toward periodicity or waviness in the surface with an average frequency of about 50 per foot. The mean amplitude of the extremes of these irregularities about the moving average of 10 was about $\pm$ 0.0017 foot.

There seems to be no way of knowing what characteristic of the roughness is pertinent to the flow qualities. It may well be that different slope and discharge ranges are affected by different roughness characteristics of the bed. The mean elevation of the rims of the pits would probably be more nearly correct than the overall average for the bed surface, and somewhat higher.

Average water surface elevations above the arbitrary datum for the low flows over this rough surface are plotted in figure 5, page 9, against the calculated depths for pure laminar flow. It may be seen that the data for the lower flows for all of the several slopes can be quite well represented by a straight line, giving an average bed elevation of 0.0092 foot at zero calculated depth. This elevation is assumed in subsequent calculations to be the effective bed elevation. At this

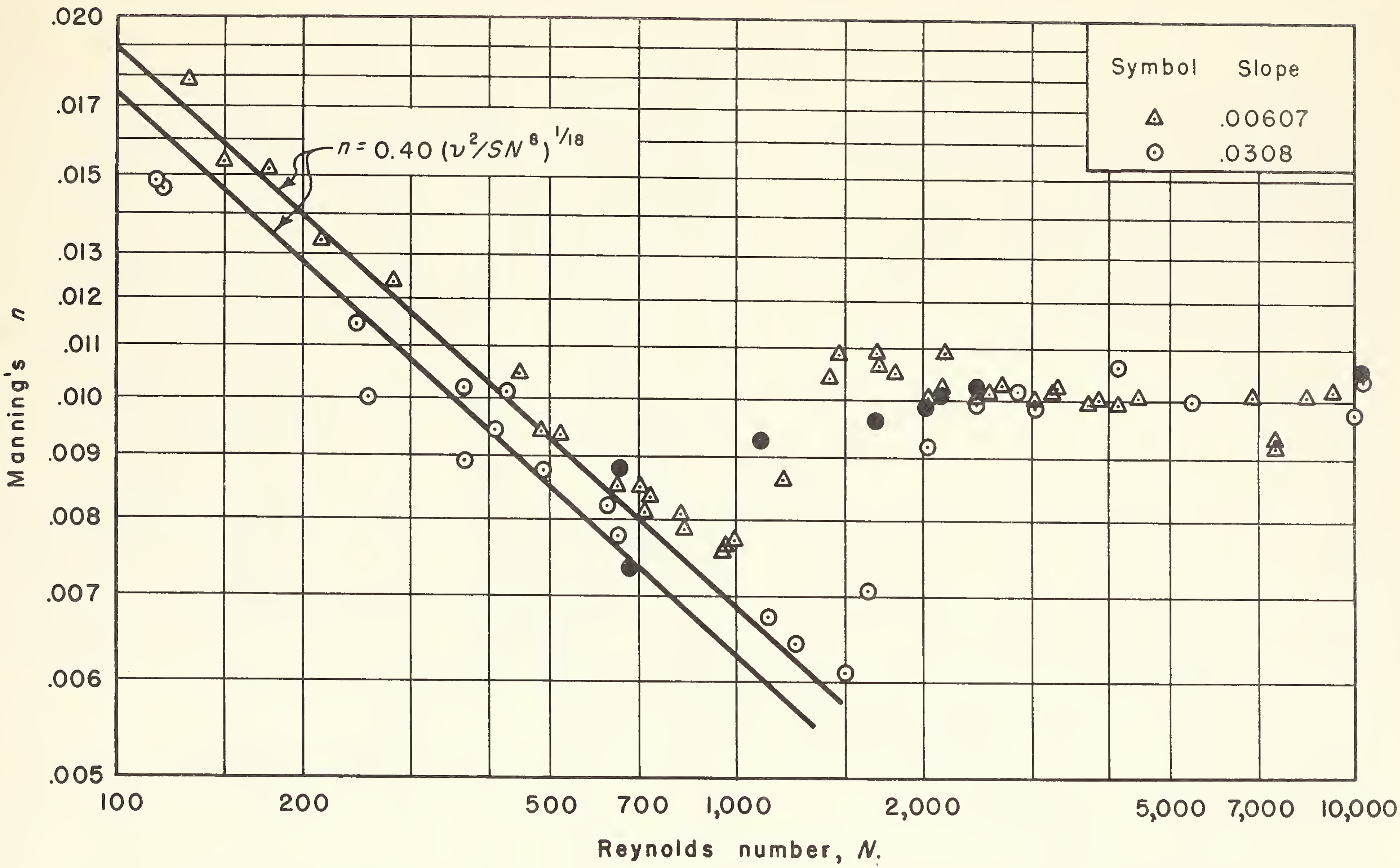

Figure 3.— Manning's n for flows on a smooth bed.

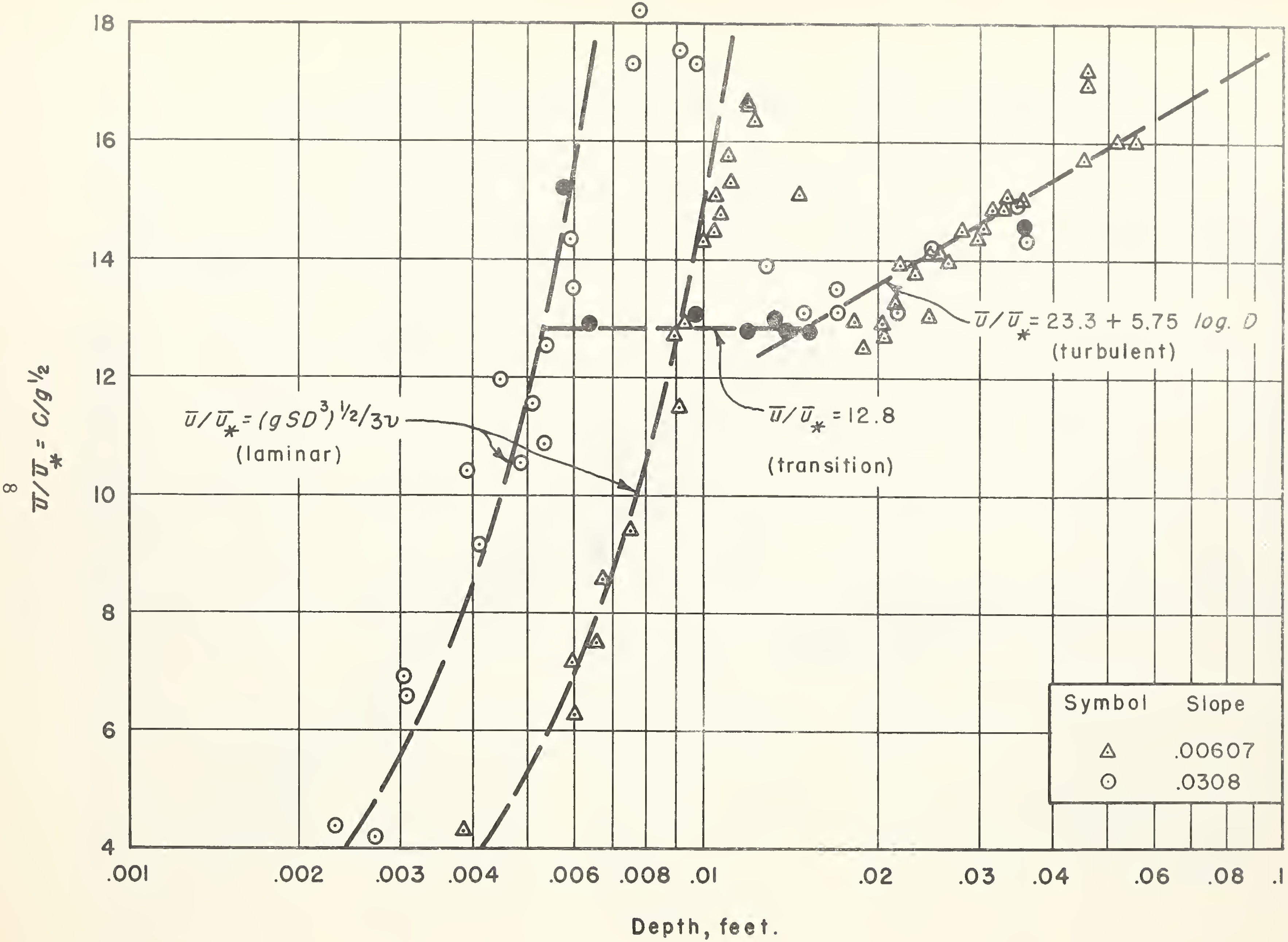

Figure 4.— Variation of the mean flow ratio with depth on a smooth bed.

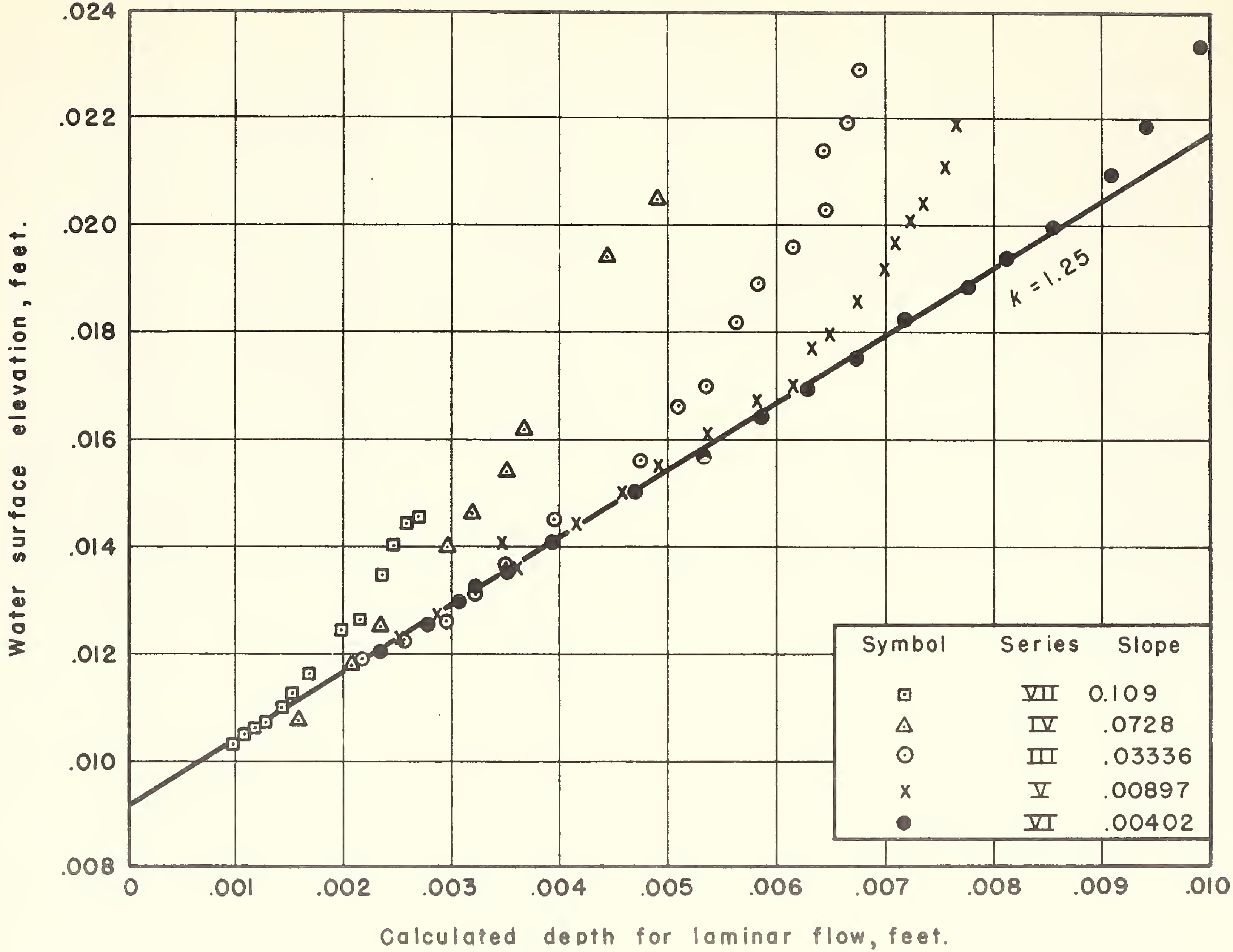

Figure 5.— Depth of sheet flow on a rough, pitted bed.

elevation the dead water storage in the depressions was equivalent to 0.0008 foot.

The slope of this line is greater than unity, being 1.25. This means that the depths of flow represented by the line are 25 percent greater than were obtained on the smooth bed at equal discharges. And, since the line is straight, the relationship between the slope, depth, viscosity, and average velocity may be considered the same as for pure laminar flow. If the depth for pure laminar flow is denoted by

$$D' = 0.454\left(\frac{q\nu}{S}\right)^{1/3} \tag{12}$$

and

$$k = \frac{D}{D'} \tag{13}$$

where D is the depth of flow over the rough surface,

then

$$D = 0.454k\left(\frac{q\nu}{S}\right)^{1/3} \tag{14}$$

and

$$q = \frac{gSD^3}{3k^3\nu} \tag{15}$$

or

$$V = \frac{gSD^2}{3k^3\nu} \tag{16}$$

Equation (15) is a general equation for sheet flow of a type related to pure laminar flow. It is presumed to be applicable to low flows subjected to disturbances and to large retarding influences. It is chosen because of its simplicity and its demonstrated applicability in the case of the data of figure 5, page 9, for which the value of k is 1.25. Its upper limit of application within the test range for the rough pitted surface is expressed by the relation

$$NS^{2/3} = 7.5 \tag{17}$$

This is, for example, a Reynolds number of 35 on a 10 percent slope and 162 on a 1 percent slope. This relation is not known to be applicable for any condition other than the one tested. The reason for the greater depths is not specifically known but it is thought to be largely due to turbulence continuously generated at the rough bed. In later applications and possibly in this case there may have been blocking of the flow by bed projections in addition. There is also to be considered the probability that the effective distances of fluid filaments from the retarding surfaces are less than for a smooth bed with equal depth of flow.

Surface Speeds of Viscous Flow Over a Rough, Pitted Bed

Water surface velocities were determined in a few instances by measuring the speed of bronze powder floating on the surface. The surface was visibly turbulent at surface speeds exceeding 0.5 ft./sec. for all three slopes used. The measured average surface speed is compared in figure 6, page 11, with the surface speed for pure laminar flow as calculated by

$$V_s = \frac{3}{2}q^{2/3}\left(\frac{gS}{3\nu}\right)^{1/3} \tag{18}$$

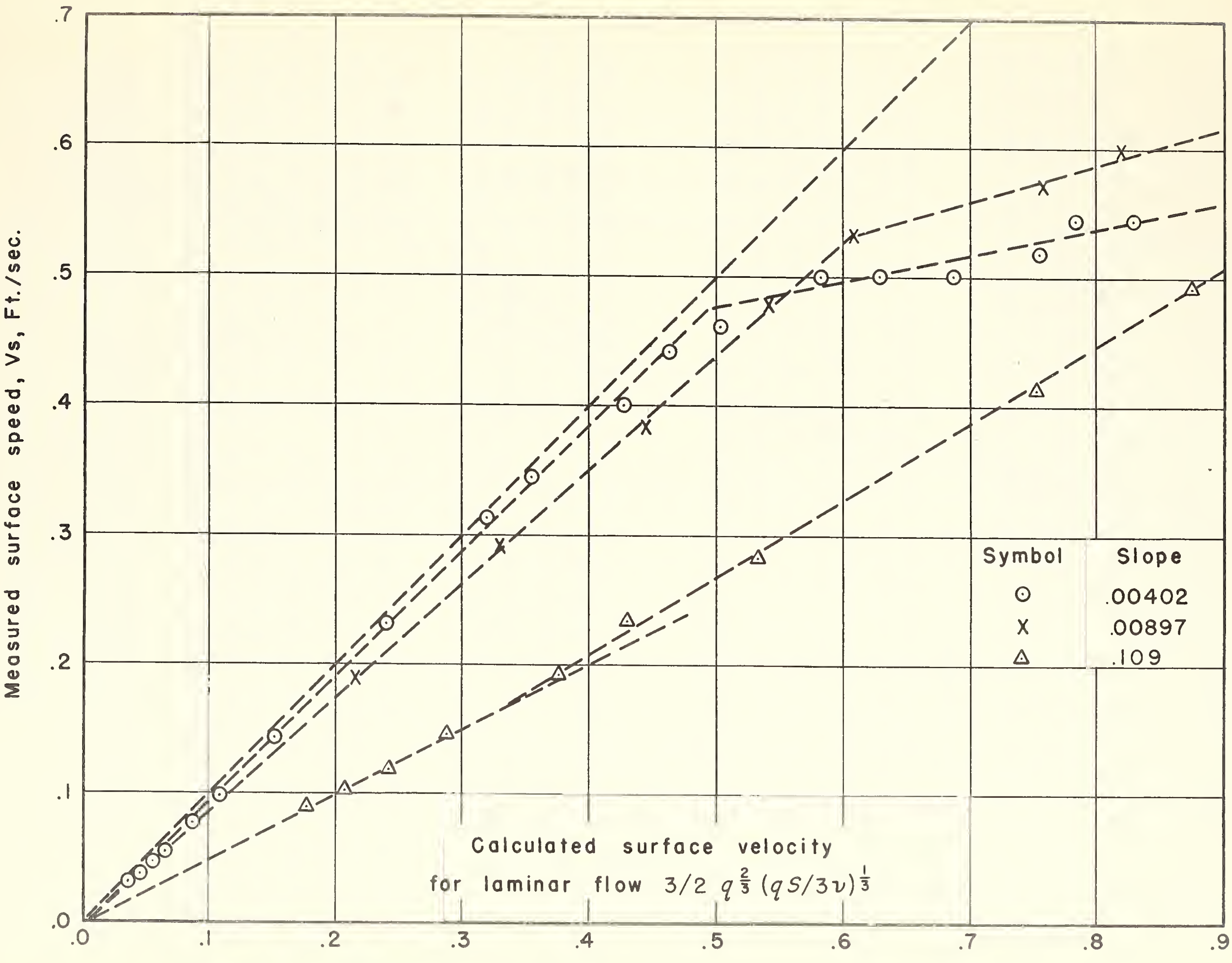

Figure 6. — Water surface speeds on a rough pitted bed

It may be seen that the measured speed is almost equal to the calculated speed for the flat slopes. There is, however, a definite tendency for the surface speed to become proportionately less than the calculated speed as the slope increases. At a slope of 0.109 with very low flows, the surface speed is one-half that for pure laminar flow at like discharges.

The compatibility of the surface speed data with those on depth of flow is not obvious. To reconcile these measurements, one must speculate about the differences in the vertical distribution of velocity in the flow, both between that on a smooth and on a rough bed and that between flat and steep slopes on a rough bed. Whereas, the ratio of surface to average velocity for pure laminar flow is 1.5, it is about 1.9 on flat slopes and 1.0 on an 11 percent slope for the rough bed. The latter values are derived from the apparent relationships depicted in figures 5 and 6, pages 10 and 11. The accuracy of the estimated value for the 11 percent slope is probably not too good because of the extremely small depths involved. A ratio of unity would indicate that the speed of the water was constant from the surface down to the bed.

It may be of interest here to point out that speeds of electrolytes or dye in laminar flow, as determined by measuring the time of occurrence of maximum concentration at some point downstream from the point of injection, are surface speeds, if suitable corrections are made for the volume of the injected fluid. More correctly, and in the case of closed conduits, maximum velocities are determined in this manner. This may be deduced by drawing a simple diagram as in figure 7, page 13. The electrolyte is introduced at T_o at A and occupies a length of channel ΔL. In time, T, the top lamina travels to point B. The other laminae travel lesser distances in accordance with their vertical position within the flow as expressed implicitly by equation (1), page 1. The average concentration, or the area of electrode in contact with the electrolyte, is proportional to the vertical thickness of the contaminated laminae. This is a maximum at B. In general ΔL is an insignificant part of L.

Turbulent Sheet Flow

The mean flow ratios for all the flows over the rough pitted surface are plotted against log $D(gDS)^{1/2}/\nu$ in figure 8, page 14, along with the data for the smooth bed. This method of plotting is not to infer that the rough surface was hydrodynamically wavy in the range of turbulent flows. It simply gives a more orderly and less confusing picture of the results than do other methods. Here again is demonstrated a tendency for a constant value of Chezy's C and the mean-flow ratio within the transition region between viscous type and turbulent flows. The mean-flow ratio within this range for the rough bed may be approximately expressed by $\bar{u}/\bar{u}^* = (\text{Const.}) - 5.75 \log S^{1/2}/\nu$. Although the data are insufficient to be conclusive, it is presumed that the value of the constant is dependent upon the roughness of the bed.

These same data for Reynold's numbers greater than 2,000 are replotted in figure 9, page 15, in a fashion to show the position and behavior of the flows relative to bed roughness classification by logarithmic analysis and Nikuradse's equivalent sand size, k_s. If the data should fall along the upper diagonal line, viscosity is a factor in the flow and the channel or bed would be considered to be hydrodynamically smooth. If the data should fall along a line parallel with but lower than the upper diagonal line, according to Keulegan the viscosity is a factor and the bed would be considered to be hydrodynamically wavy. When they fall along a horizontal line, as they nearly do, the flow is independent of viscosity and the bed is considered to be hydrodynamically rough. Rouse (5) shows that for pipe flow a transition region is between the two diagonal lines and that all data falling well below the lower line $(\bar{u}^*/\nu = 70/k_s)$ would represent hydrodynamically rough boundary surfaces. There may be some question about the channel being sufficiently long to obtain uniform flow for the larger discharges.

Although the conventional coefficient of 5.75 for the logarithmic term has been used, the data suggest a mean value of about 6.2, giving for the turbulent flow over the rough, pitted surfaces

$$\frac{\bar{u}}{\bar{u}^*} = 17.8 + 6.2 \log D$$

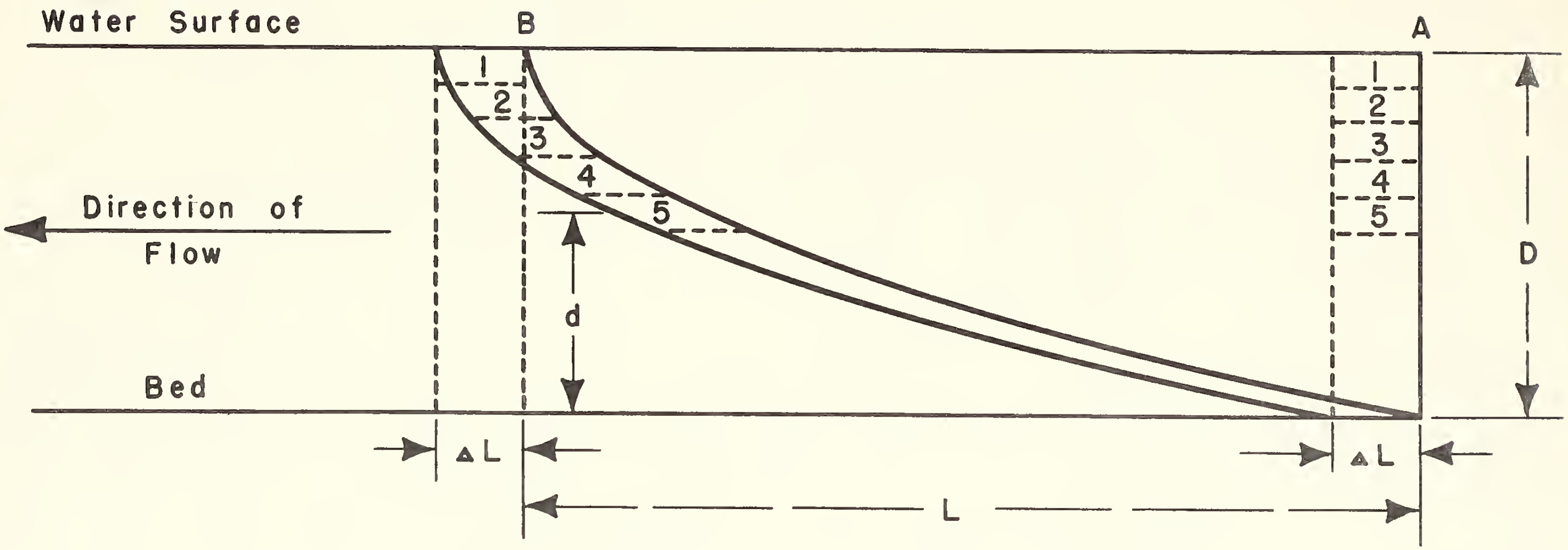

Figure 7. - Illustration of the flow of dye or electrolyte in laminar flow.

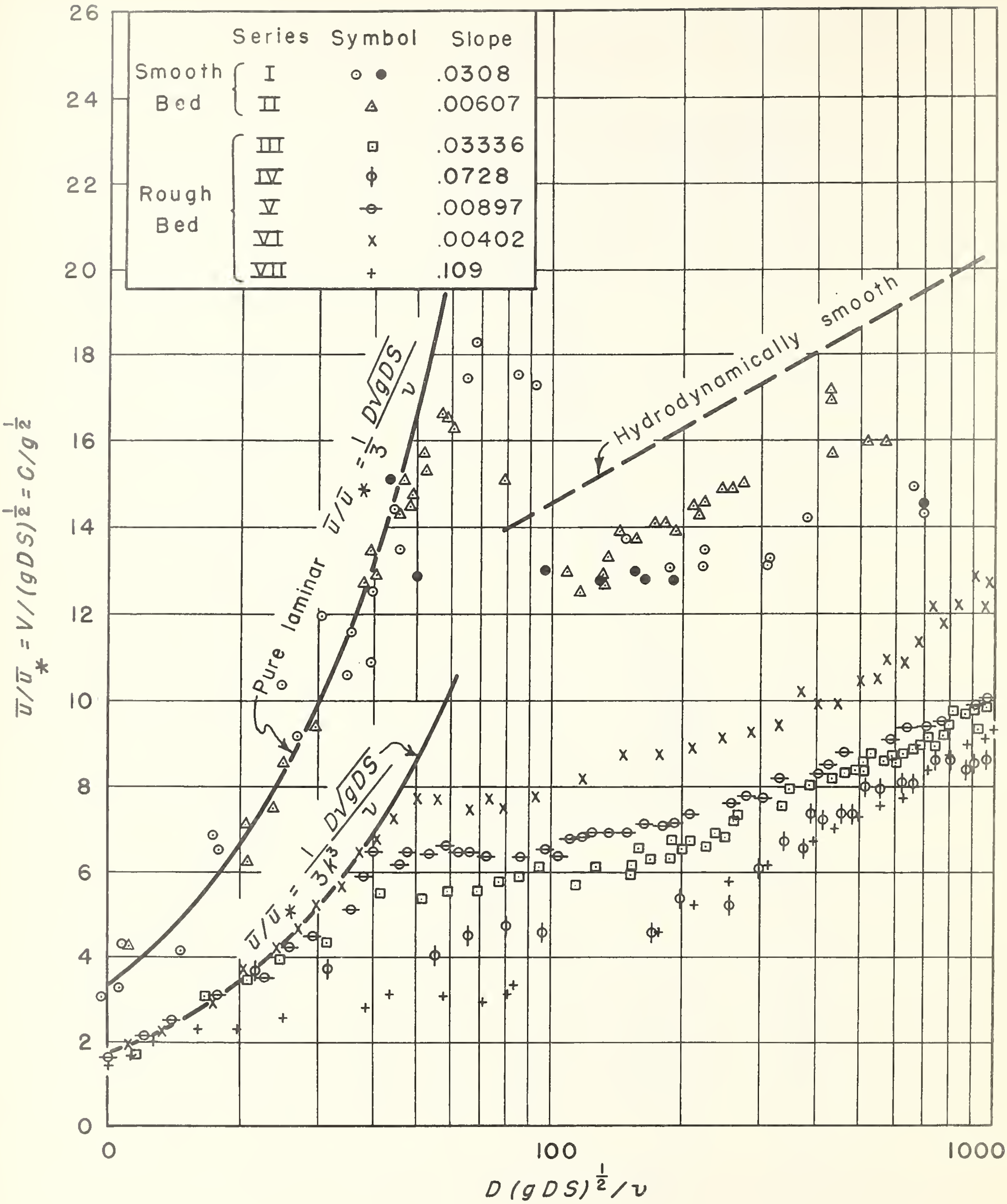

Figure 8.—Variation of the mean flow ratio with the Reynold's number based upon boundary shear velocity.

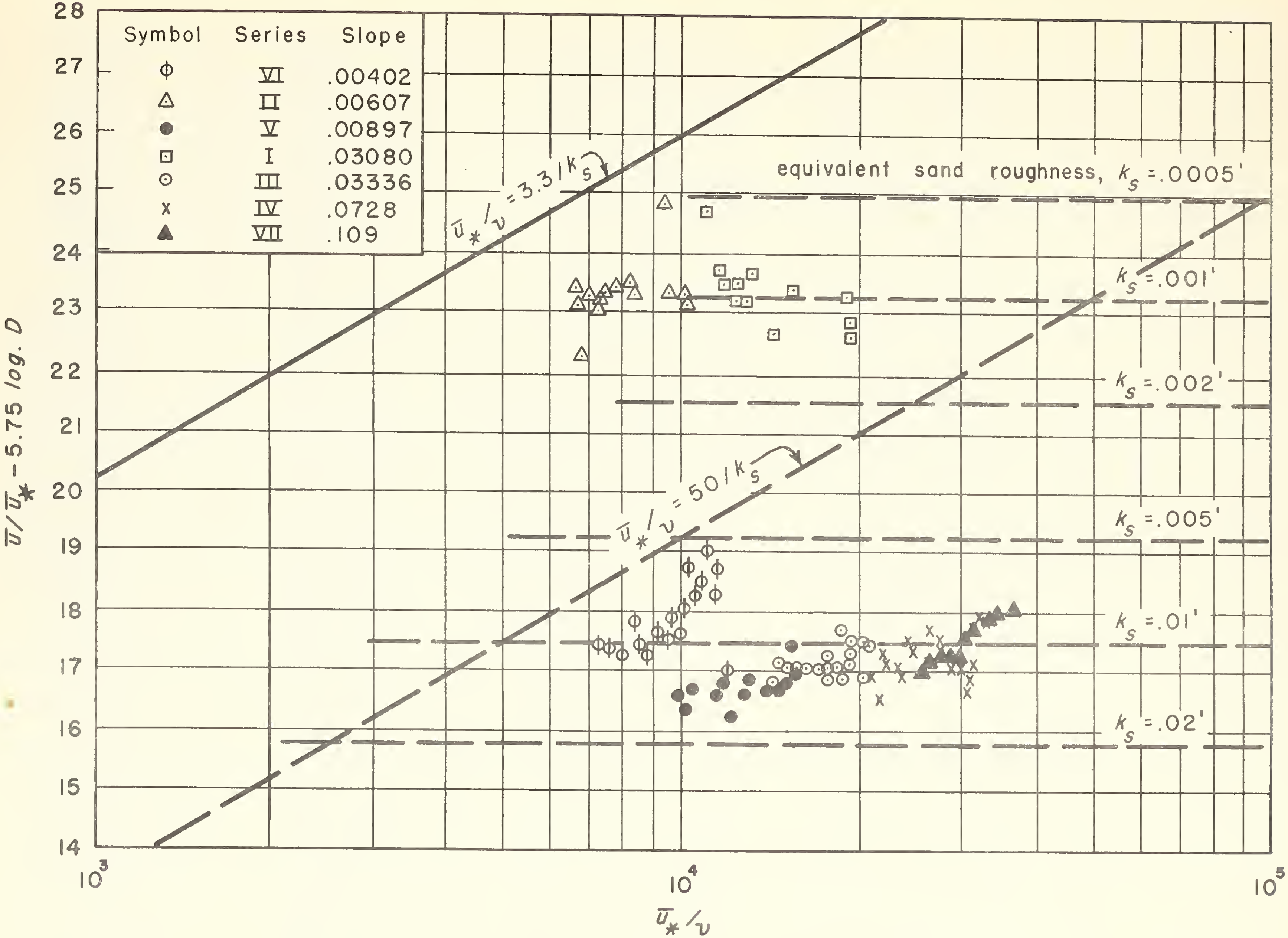

Figure 9.— The positions of the turbulent flows in respect to the several ranges in viscosity influence, and to Nikuradse's sand grain size.

The value of the coefficient appeared to increase with increasing slope.

A parallel curve approximately representing the points of departure from viscous type to transition type flow is expressed by

$$\frac{\bar{u}}{\bar{u}*} = 19.6 + 6.2 \log D$$

This indicates that the depth of flow was approximately doubled through the transition range from viscous to fully turbulent flow.

The Effect of Rainfall Impact on the Depth of Viscous Flow

An attempt was made to evaluate the effect of rainfall impact upon low flows over the rough bed. The spray from type-F nozzles,[2] mounted at bed elevation and operated at 35 lbs. per square inch pressure, was caused to fall on a small initial flow. The extra water from the spray caused an increase in depth in the channel and an increase in the discharge, varying from zero at the upper end to a maximum at the lower end. Assuming that the spray was uniformly applied along the channel, the average increase in depth for the increased discharge was calculated in conformance with the depth-discharge relationship obtained in the absence of spray-drop impact. The extra depth credited to spray-drop impact was obtained by deducting the calculated increase from the measured increase.

The measured increase was an average of two determinations. By suitable flow and time measurements the deficiency in discharge between the time of beginning of the spray and some later time when the flow was constant was determined. This value for increased storage was averaged with the similarly determined excess in discharge following an abrupt end of spray application. Fifteen complete observations were made using different slopes, initial flows, and spray application rates. The excess depth ranged from 8 to 28 percent of the average depth, averaging 17 percent. No consistent variation in relative increase with slope, flow, or spray rate was noted. Assuming the applicability of equation (15), page 10, for this flow condition gives a value of $k = 1.46$ for the rough pitted surface with spray-drop impact. It should be noted that the kinetic energy of the spray at impact was less than that of the spray of the type-F artificial rainfall apparatus due to the fact that the height of fall of the spray was 2 feet less.

VISCOUS OVERLAND FLOW

Development of Field Formulae

Flow conditions in nature, out on the land, are generally different than those tested in the laboratory, and highly variable. The relationship of the variables, expressed by equation (15) is apparently satisfactory for the limited laboratory conditions, but does not necessarily represent the behavior of the flow in the field. Additional evidence is needed in support of its applicability. The results of tests with artificial rainfall will be used in order to more nearly approach field conditions, but first it will be advantageous to develop expressions for the flow in field language and symbols.

The flow in cubic feet per second from a sloping strip 1 foot wide with steady runoff resulting from uniform rainfall is

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[2] The type-F nozzle was developed by V. D. Young, Soil Conservation Service, in 1939 expressly for use in work requiring simulated rainfall. A brief reference to its development is given in a discussion of Wilm's (7) paper in the 1943 Transactions of the American Geophysical Union. See Kelly (3) for a description of the performance of the type-F apparatus.

$$q = \frac{rL}{43200} \qquad (19)$$

where r = runoff rate, inches per hour
L = length of slope (or strip), feet.

The bulk Reynolds number is

$$N = \frac{VD}{\nu} = \frac{rL}{43200\,\nu} \qquad (20)$$

For disturbed viscous flow in accordance with equation (15), page 10, the average velocity at L distance downslope from the upper boundary of the strip is

$$V = \frac{rL}{43200D} = \left(\frac{rL}{43200}\right)^{2/3} \left(\frac{gS}{3k^3\nu}\right)^{1/3} \qquad (21)$$

and the depth is

$$D = k\left(\frac{3\nu rL}{43200gS}\right)^{1/3} = k\left(\frac{3\nu^2 N}{gS\,\nu}\right)^{1/3} \qquad (22)$$

By integration the average depth of flow, $\overline{D}$, over the whole area is found to be $\frac{3}{4}D$.

The Measurement of Water Depth on a Sprinkled Plot

The average depth of detained water on the surface of a plot of ground being sprinkled at a constant rate with artificial rain can be quite closely determined. It is essential for an accurate determination that both the rainfall rate and the infiltration rate be sensibly constant. The method employed involved two periods of application. Rain was applied in the first period until a near constant infiltration rate was attained, as evidenced by a near constant measured flow from the foot of the plot. The rain was stopped for an interval of time just sufficient for the surface storage to be dissipated through runoff, infiltration, and evaporation. The deficiency in the volume of runoff between the time of the beginning of the second period of application and some later time when the flow again became constant was the volume of stored water on the plot surface. This volume divided by the plot area gave the average depth on the plot.

In making such determinations it is important to remember that because of viscous flow and depression storage there may be a considerable depth of water remaining on the plot after the discharge has become very small. Another possible complication arises when an appreciable portion of the flow occurs below the surface. This might easily be the case with transposed vegetation, like sod, laid on a relatively impermeable subsurface.

Bare Soil Tests on a 50-foot Tilting Plot

A 50-foot tilting plot at Auburn, Ala., was utilized in obtaining the results given in table 1, page 19. A type-F artificial rainfall apparatus was mounted on the plot. Runoff was measured from an area 6 feet wide by 47.5 feet long with sheet metal boundaries. The soil was Decatur clay loam from north Alabama. It was bare and had been considerably compacted. The depths of water on the plot surface were determined in the manner described and also by measuring the excess or deficiency in discharge while changing the slope of the plot during the progress of the test. Soil erosion occurred throughout the test.

The results of this test are shown in figure 10, page 20, by the group of points designated A. The depths of water on the plot are compared with the depths for like slopes and discharges in pure laminar flow. Here again a straight line passing through zero calculated depth can be drawn to adequately represent the data. The slope of this line is the value of k in equations (15) and (22), pages 10 and 17, and is 2.8. In other words, the depth of flow on bare, eroding Decatur clay loam subjected to the spray of the type-F artificial rainfall apparatus is 2.8 times greater than for pure undisturbed laminar flow. The runoff water was exceedingly turbulent, yet it appears, in a modified fashion, to have behaved like viscous flow.

A second test was made on this same plot several months later. The plot was used in the intervening interval for trial observations of steady flows without artificial rainfall. These flows caused pronounced rilling; and, even though the surface was raked and leveled, the rills reformed in the early part of the second test with artificial rain and grew progressively during the test.

The formation of rills and gullies, in effect, shortens the distance, L, of overland flow. Instead of beginning at the upper boundary and increasing progressively until it reaches the lower boundary, the flow originates at the upper boundary, or the divides between rills, and ends, in part, in concentrated flow uphill from the lower boundary. The concentrated flows in these channels may be much greater in depth than the average, but the relative area of the land that is covered by these flows is sufficiently small for this factor to be outweighed by the effect of shortening the distance of overland flow. The mere presence of rills or flow concentrations rather than their depth seems to be the more important influence.

Water was introduced at the top boundary of the plot during a portion of the time in the second test. Under this condition the plot represents the lower portion of a longer plot. The runoff rate for steady flow is the difference between the spray application rate, i, and the infiltration rate, f. The flow per foot width, q, introduced at the upper boundary represents the runoff per foot width from a length of plot,

$$L_1 = 43200 \frac{q}{r} = 43200 \frac{q}{i-f} \qquad (23)$$

The total length of overland flow, L_2, is then, in this case, $L_1 + 47.5$. By integration the average depth $\overline{D}$ for laminar flow for the lower 47.5 feet of a longer plot is found to be

$$\overline{D} = \frac{3}{4} \left(\frac{3 \nu r}{43200 gS} \right)^{1/3} \left(\frac{L_2^{4/3} - L_1^{4/3}}{47.5} \right) \qquad (24)$$

For a short period of time during the test, there was inflow at the upper boundary without artificial rain on the plot.

The essential results of the second test are given in table 2, page 19. They are represented in figure 10 by the line marked B which has a slope or value of k, of 2.2. Aside from the possibility of errors in the depth determinations, the apparent reason for the difference in the value of k between tests 1 and 2 is the marked presence of flow concentrations in the second test.

There appears to be a slight tendency for the average depth at high flows to be greater than for a value of k = 2.2. This may be due to the flow exceeding the upper limit of applicability of equation (22) or to the fact that increased depths and velocities tend to minimize the plot irregularities that produce concentration. Rather high Reynolds numbers were reached in some of the flows. The Reynolds number is greatest at the foot of the plot when rainfall is applied.

TABLE 1.--*Water depths on bare, Decatur clay loam, test 1*

Slope	Runoff rate	Average depth	Maximum Reynolds Number	Average depth for laminar flow
%.	*in./hr.*	*ft.*		*ft.*
10	1.52	0.0055	129	0.00205
5	1.54	.0069	131	.00259
2.5	1.56	.0091	132	.00328
2.5	3.32	.0118	280	.00421
2.5	1.55	.0092	131	.00327
5	1.76	.0078	149	.00270
10	1.73	.0063	147	.00214
20	1.75	.0051	148	.00172
10	1.67	.0061	141	.00211

TABLE 2.--*Water depths on bare, Decatur clay loam, test 2 (slope = 0.03)*

Time	Runoff rate	L_2	Maximum Reynolds Number	Average depth	Average depth for laminar flow
min.	*in./hr.*	*ft.*		*ft.*	*ft.*
-27-0	1.25	47.5	136	0.0063	0.00264
0-12	1.64	47.5	182	.0063	.00289
16-33	1.70	47.5	186	.0063	.00292
33-48	3.72	47.5	405	.0087	.00380
48-63	3.70	66.1	570	.0116	.00485
63-78	3.68	83.3	722	.0132	.00542
78-90.17	1.80	116.7	517	.0113	.00494
90.17-105.25	(No rainfall)		295	.0093	.00463
105.25-114	1.80	116.7	517	.0107	.00494
114-129.5	1.80	83.4	364	.0093	.00428
129.5-144	1.80	47.5	198	.0066	.00298

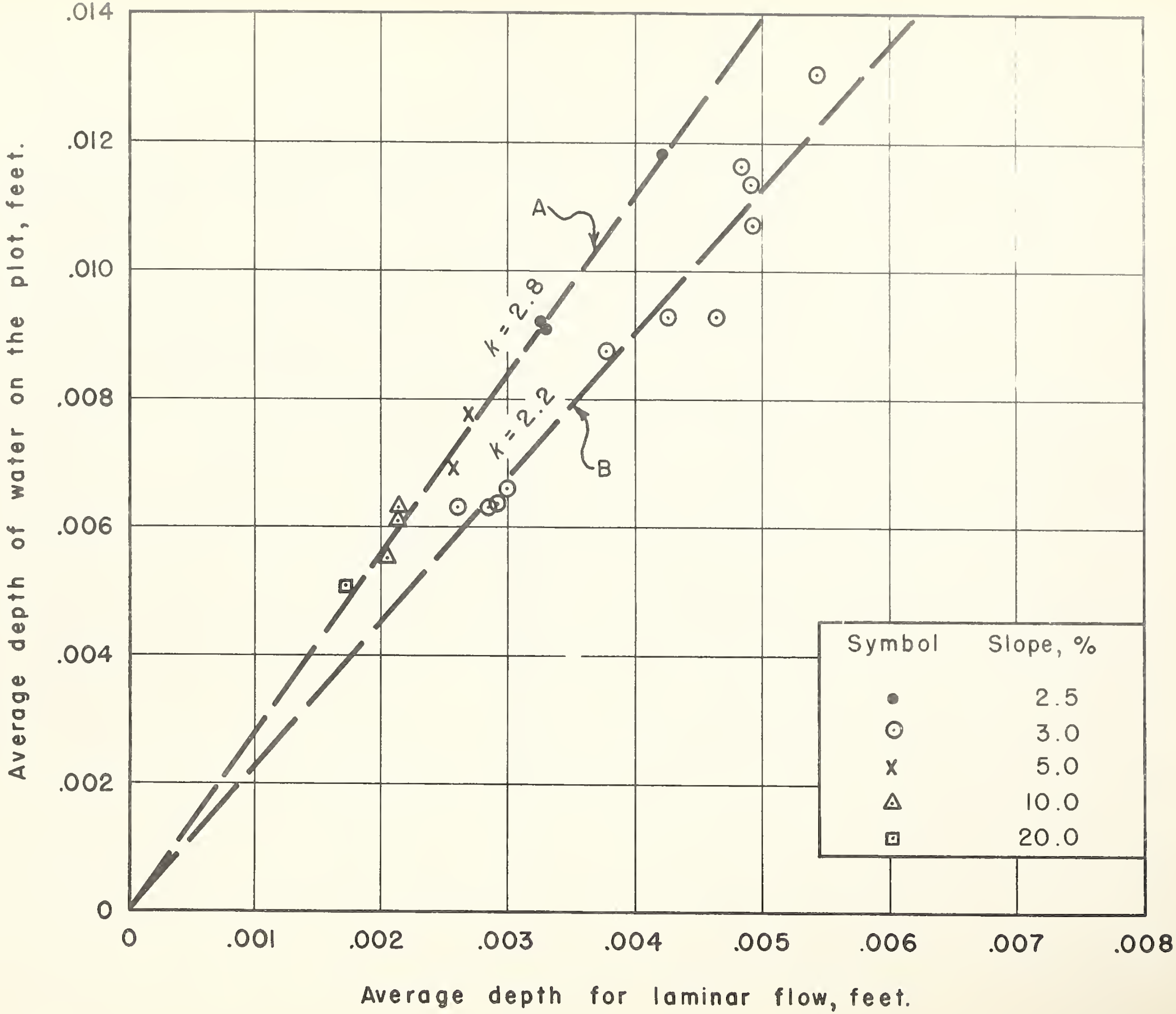

Figure 10.— Average depths of water on bare, eroding, Decatur soil.

Bare Soil Tests with Cold and Warm Water

Table 3, page 24, gives some results obtained in another series of artificial rainfall tests, using the type-F artificial rainfall apparatus on bare Chesterfield loamy sand. Plots A and B were 6 feet wide by 12 feet long. Plots C and D were 6 by 24 feet. All plots were on a 6 percent slope. Three tests were made on plot D and one on each of the others. A straight-edge was used to smooth off the soil prior to each test.

Average depths of water on the plot surface are shown and are compared by means of ratios (values of k) with the calculated depths for laminar flow which would have prevailed under the test conditions in the absence of surface roughness and raindrop impact. Assuming uniform depth across the plot, the highest Reynolds number was 255, occurring at the foot of the plot in test D-3. Soil erosion occurred in all tests.

The depth ratios were grouped according to rate of runoff, water temperature, and plot length. The average for each group is shown in table 4, page 24. The variation in average depth ratio between the long and short plots can be explained by the fact that flow concentrations were observed to exist on the lower end of all the long plots, especially in the second and third tests on plot D. The observable concentrations varied in number from one to a dozen across the plot and from short lengths up to half the plot length. They varied in extent during each test and from test to test and were more of the nature of gradual variations in depth across the plot rather than steep-sided, sharply defined channels.

The higher depth ratios for the high runoff rates on the long plots are compatible with the idea that flow concentration is the main factor involved in the variation of the depth ratios. The higher velocities and greater depths prevailing during high runoff tend to reduce the relative depth variation across the plot.

The depth ratios for the warm water tests in all six pairs of tests were consistently greater than for the cold water tests. Disregarding the small likelihood of a systematic difference in plot surface roughness, leaves only the influence of viscosity differences to account for the differences in ratios. A plausible explanation is that the turbulence created by the rough surface and by the impact of spray drops causes an increase in depth. The turbulence is damped to a greater extent by the relatively high viscosity of the cold water.

Tilting Plot Tests with Lespedeza Cover

One of the 50-foot tilting plots was also used in determination of overland flow depths on an excellent stand and growth of Korean lespedeza. The vegetation was green and nearly mature. The results are shown in figure 11, page 23. The slope of the line representing the data is 9.1 indicating that the depth of water that was associated with the flow was 9.1 times greater than for pure laminar flow. A measured depth of 0.006 (0.07 in.) is indicated at zero calculated depth.

Any attempt to explain this initial storage involves some speculation, yet there were three reasons for such a result to be obtained. First, due to the unanticipated difficulties, the time interval between the end of the first application of water and the beginning of the second was 3-1/2 hours instead of a fraction of 1 hour as it should have been. The vegetation was still damp at the beginning of the second rainfall application, but there undoubtedly occurred an appreciable amount of evaporation and stem flow during the interval. Therefore, a portion of the potential canopy interception needed to be satisfied. Second, a straight line extrapolation of infiltration rate back to the beginning of the second period was probably an underestimation for the first few minutes. In other words, there may have been more water going into the ground than was estimated. Third, there was known to be a few clean-cut depressions made by the legs of a ladder. The stored water in these holes and in other less well-defined depressions that probably existed could not contribute to the flow.

Tests with Other Covers

Some artificial rainfall tests with the type-F apparatus were made in Anacostia Park, D. C., in 1939. The vegetation was largely bluegrass but included a small amount of crab grass. Three sites were tested, but the slopes were measured on only two. Water-surface elevations on plot 1 were determined by point gages referenced to an intensive survey of plot surface elevations. The tests on plot 2 were made in a manner suitable for analysis by the method heretofore described. The plots were 6 feet wide by 12 feet long. The essential data are shown in table 5, page 24. The k values for bluegrass from these tests are 10.2 and 8.9.

Tests at Auburn in 1947 on Chesterfield soil, using the type-F apparatus and plots 6 feet wide by 6-1/2 feet long, gave values for k of 2.35 and 5.4 for bare soil and broom sedge cover, respectively.

The Growth of Runoff and Time of Concentration for Uniform Rate of Supply for Runoff

Since numerous attempts have been made to determine the relationship of variables affecting overland flow by studies of runoff growth curves obtained in artificial rainfall tests, the equations of this report are applied. The schematic drawing in figure 12, page 25, shows the flow condition at time t on an ideal plot during the growth of surface detention.

From equation (22), page 17, the flow across the lower boundary at time t in terms of runoff rate is,

$$r = 43200 \frac{gSd^3}{3k^3 \nu L} \tag{25}$$

and the rate after attainment of steady or final flow is,

$$r_f = 43200 \frac{gSD^3}{3k^3 \nu L} \tag{26}$$

Then the ratio of runoff rate at time t to the final rate is

$$\rho = \frac{r}{r_f} = \left(\frac{d}{D}\right)^3 = \left(\frac{3d}{4D}\right)^3 = 0.42 \left(\frac{d}{D}\right)^3 \tag{27}$$

But, with depth in feet, time in seconds and rainfall and infiltration rates in inches per hour,

$$d = \frac{(i-f)t}{43200} = \frac{r_f t}{43200} \tag{28}$$

Then

$$\rho = 0.42 \left(\frac{r_f t}{43200D}\right)^3 \tag{29}$$

Equation (29) is plotted in figure 13, page 26, along with measurements from selected tests to demonstrate its applicability. The value of D_o is the sum of the depths of water required at the beginning of rainfall application to satisfy the demands of interception by vegetation, infiltration in excess of the rate at the attainment of the final runoff rate, and that part of depression storage through which downhill flow does not occur. The value of t_o is the length of time required to obtain D_o.

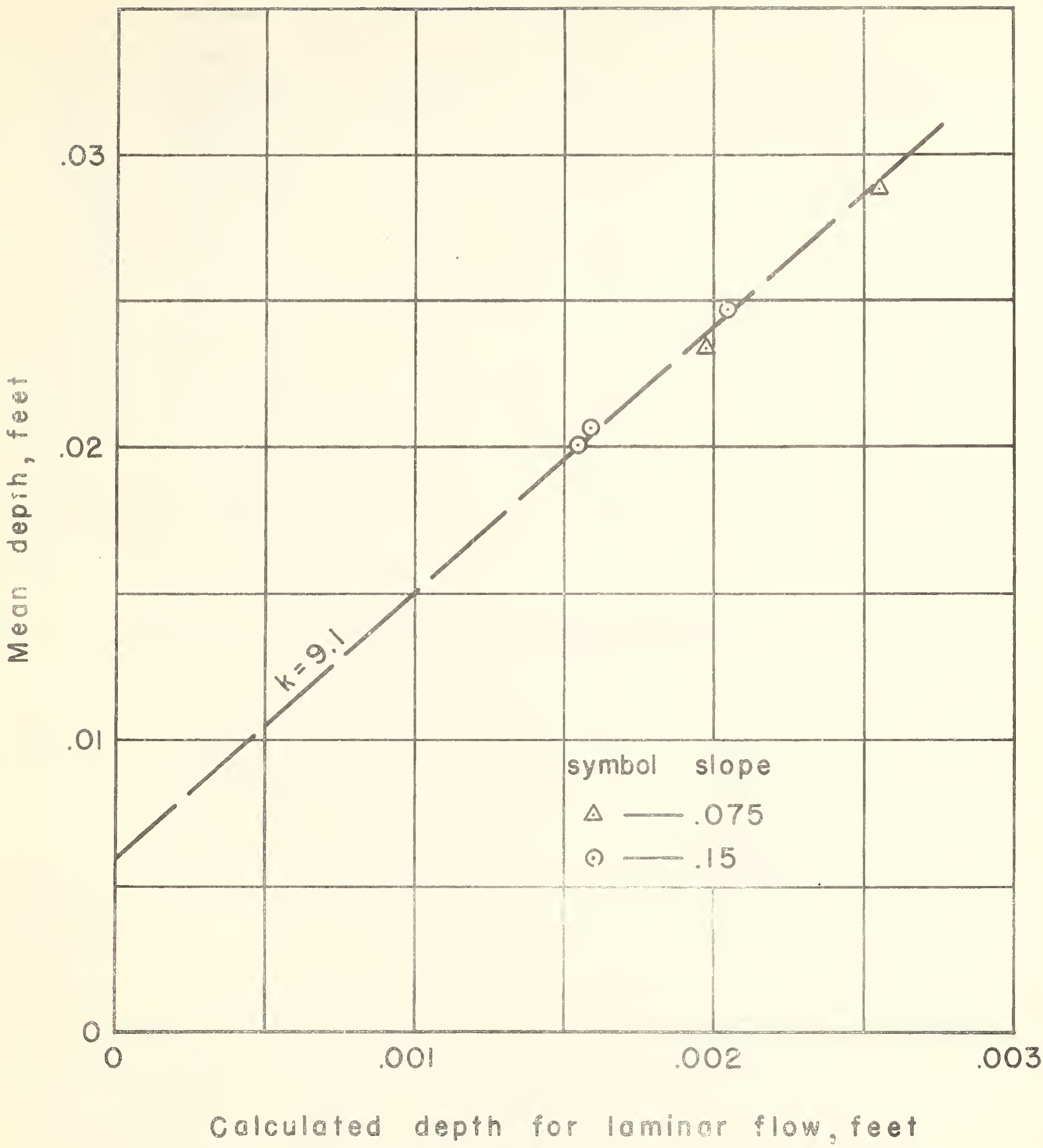

Figure 11.—Average depths of water with Korean lespedeza ground-cover.

TABLE 3.--*Average water depths on bare Chesterfield loamy sand*

Plot and test No.	Plot length	Runoff rate	Water temperature	Average depth	*D/D'* = *k*
	ft.	*in./hr*	°C	*ft.*	
A-1	12	1.76	19.2	0.0029	1.86
		3.73	19.2	.0034	1.74
B-1	12	1.63	28.6	.0027	1.92
		3.47	31.8	.0036	2.06
C-1	24	1.88	19.8	.0032	1.65
		3.89	19.8	.0042	1.68
D-1	24	1.61	29.1	.0031	1.79
		3.44	32.3	.0041	1.87
D-2	24	1.94	21.8	.0028	1.46
		4.01	21.5	.0041	1.64
D-3	24	1.83	33.0	.0029	1.64
		3.73	33.0	.0039	1.74

TABLE 4.--*Average depth ratios for low and high runoff rates, for short and long plots and for cold and warm water*

(Chesterfield Loamy Sand)

Condition	Plot length	
	Short	Long
Low runoff rate	1.89	1.63
High runoff rate	1.90	1.74
Cold water	1.80	1.61
Warm water	1.99	1.76

TABLE 5.--*Water depths on bluegrass (Anacostia Park, D. C., 1939)*

Plot	Slope	Runoff rate	Average depth	*k*
		in./hr.	*ft.*	
1	0.006	1.3	0.029	
		3.0	.039	10.2
2	.014	1.66	.022	8.9

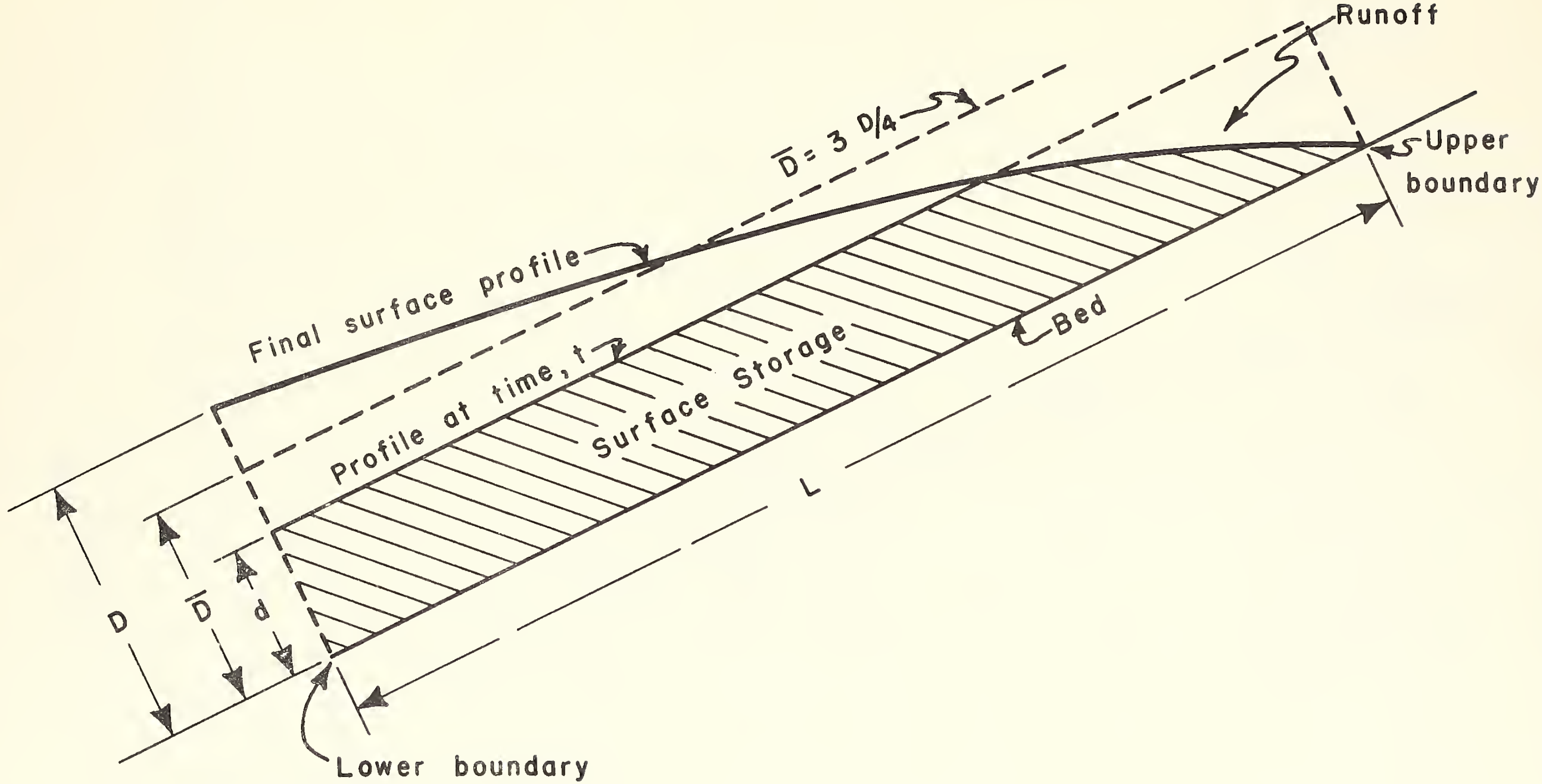

Figure 12.— Schematic representation of water on an ideal plot during the growth of runoff.

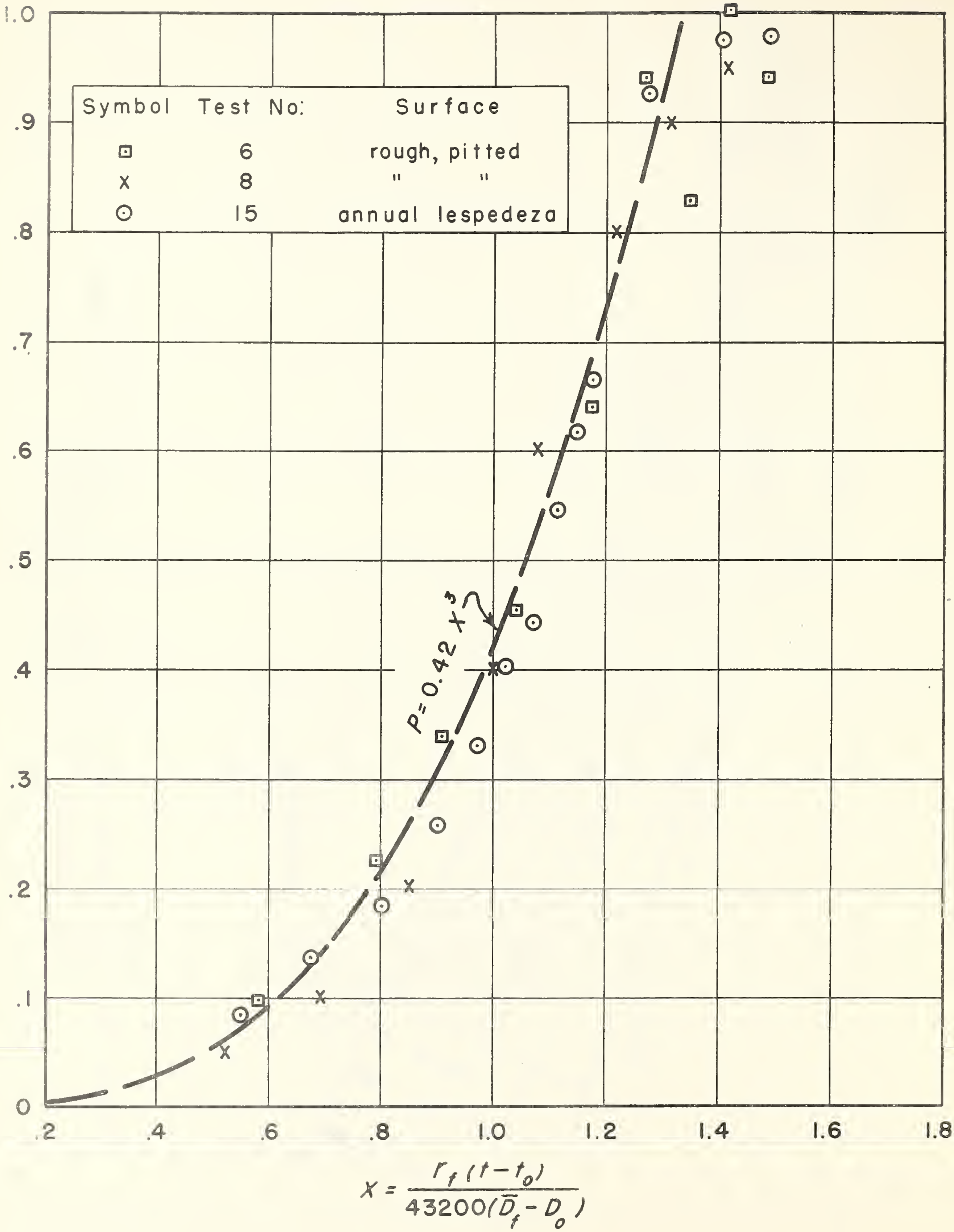

Figure 13.— The growth of runoff from sprinkled plots.

Thus

$$t_o = \frac{43200\ D_o}{i-f}$$

The value of D_o and the reasons for its existence are previously discussed for the test on annual lespedeza. The long plots with the rough, pitted surface had deadwater, or depression storage that did not contribute to downhill flow, in the amount of 0.0010 foot as determined in a manner similar to that for the lespedeza. This compares with 0.0008 foot actually measured for the rough, pitted surface of the laboratory channel, that was similarly produced but with a spray of lower energy at impact.

When $\rho = 1$, the time of concentration, t_c, is

$$t_c = \frac{4}{3}\ \frac{43200\bar{D}}{r_f} \qquad (30)$$

$$= K\left(\frac{43200}{r_f}\right)^{2/3}\left(\frac{3\nu L}{gS}\right)^{1/3} \qquad (31)$$

In actual tests the runoff rate is found to be a little less than the final value at this time; usually within 10 percent.

The greatest difficulty in studies like these is in the quantitative determination of the demands upon the applied rainfall. The difference between the amount of rainfall applied and the amount of runoff up to a given time is usually the sum of infiltration, interception by vegetation, depression storage, and surface detention. It is always necessary to estimate infiltration by extrapolation of rates determined later in the test. The degree of satisfaction of canopy interception at the beginning of the test is a question; and, to make the picture a little more complex, it appears from the results of the Anacostia Park tests and from visual observations in other tests that the average depth of flowing water is not equal to the depth of water temporarily detained but includes also a portion of that considered to be depression storage.

Generally, then, there is required an initial period of rainfall to satisfy interception, infiltration in excess of the estimate, and that portion of depression storage through which effective downhill flow does not occur. Theoretically this initial period may be determined as in figure 14, page 28, by plotting the cube root of the runoff rate (or of ρ) against time after beginning of rainfall. The intersection of the straight line, representing the data, with the time axis gives the initial period. And, since

$$\rho^{1/3} = \left(\frac{r_f}{43200}\right)^{2/3}\left(\frac{gS}{3\nu L}\right)^{1/3}\frac{t}{k} \qquad (32)$$

the slope of the line is inversely proportional to k, with other factors the same.

The procedure of figure 14 seems good, but in practice does not give good results unless the plots are long and have uniform slope, infiltration capacity, and depression storage throughout. Storage in collecting troughs and measuring flumes becomes an appreciable factor for short plots, low runoff rates, and high rates of change in runoff.

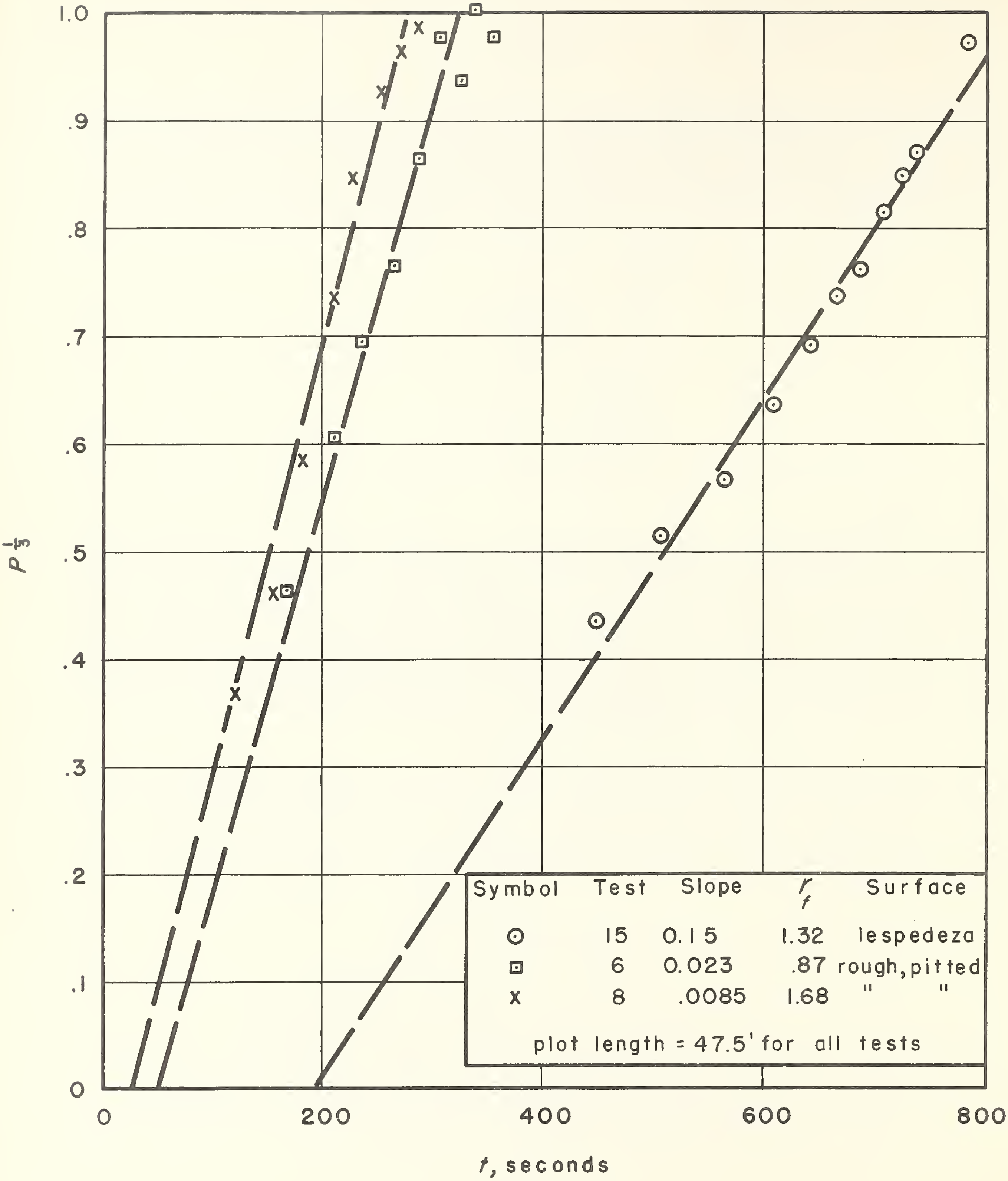

Figure 14.—The time of beginning of surface detention and the growth of runoff on sprinkled plots.

SUMMARY

Table 6, page 30, is a summary of the several determinations of the depths of flowing water under differing surface and rain impact conditions expressed as values of k in equations (14), (15), (16), (21), and (22), pages 10 and 17. These data are exceedingly limited but do cover a considerable range in conditions. The plot lengths ranged up to 47.5 feet. The k values applicable to greater flow distances or to whole fields should be expected to be somewhat lower because of the tendency for concentration of the runoff into rills, gullies, and other natural drainageways. The problem in application here is the determination of the average distance of overland flow before major concentration occurs. This is undoubtedly dependent upon slope, cover, and tillage practices. The data should be expected to apply only to relatively smooth land surfaces.

In the case of terraced fields the distances of continuous overland flow cannot exceed the widths of the terrace intervals. In a field that is terraced in accordance with the present recommendations of the State of Alabama (which differ little from most other southeastern specifications) and with surfaces sufficiently plane for application of equation (22), the average depth of overland flow is approximately inversely proportional to the square root of the land slope. In any case it is proportional to the cube root of the runoff rate; and for these conditions may be expressed in inches by the formula

$$\overline{D}\,(\text{inches}) = 0.0057k\frac{r^{1/3}}{S^{0.55}} \tag{33}$$

A few calculated values for average depths of overland flow by equation (33) are given in table 7, page 30, in order to obtain a rough notion of the magnitude of this type of temporary storage and its variation with land slope and runoff rate. Deviations from the viscous flow formula because of long slopes or high runoff rates tend to compensate the effect of the always present tendency for flow concentration. These two influences are neglected in the calculations. It should be emphasized that these are calculated values and are not a result of field size measurements. The depths shown are for laminar flow ($k = 1$). For bare soil they should be multiplied by about 2, and for a good cover, like bluegrass, by about 10.

ACKNOWLEDGMENTS

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TABLE 6.--*Measured values of k in the viscous flow equation (or the ratios of measured to theoretical depths)*

Condition	Measured values of *k*
Smooth surface	1.00
Smooth surface with traveling waves	1.08
Mortar surface, rough and pitted	1.25
Mortar surface, rough and pitted with water-drop impact (in laboratory)	1.46
Mortar surface, rough and pitted with impact of type-F apparatus spray	1.72
Bare soil with artificial rainfall[1]	1.5 to 2.8
Good broom sedge cover with artificial rainfall[1]	5.4
Excellent stand of mature Korean lespedeza with artificial rainfall[1]	9.1
Bluegrass with artificial rainfall[1]	8.9 and 10.2

[1]Artificial rainfall produced by the type-F artificial rainfall apparatus.

TABLE 7.--*Calculated average depths of overland flow for smooth, terraced fields and laminar flow, inches*[1]

Runoff rate in./hr.	Land slope, Percent				
	1	2	4	8	12
1	0.072	0.049	0.034	0.023	0.018
2	.090	.062	.042	.029	.023
4	.114	.078	.053	.036	.029
8	.144	.098	.067	.046	.037

[1]Multiply table values by 2 for bare soil and by 10 for good cover.

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C = Chezy's coefficient

d = the distance from the bed of the channel to a point within a flowing liquid

d = the depth of flow at the lower boundary of an area during the growth of runoff

D = the depth of flow

$\overline{D}$ = the average depth of water on a sprinkled area

$\overline{Do}$ = the depth of applied water as determined by the product $t_o(i-f)$

f = resistance factor in the Weisbach formula

f = infiltration capacity, inches per hour

g = acceleration due to gravity

i = rainfall rate, inches per hour

k = the ratio of depth of viscous flow to the depth for pure laminar flow

k_s = Nikuradse's sand grain size

n = Manning's retardance factor

$N = VR/$ = Reynolds number

q = rate of flow per unit width

r = radius of a circular conduit

r = runoff rate, inches per hour

r_f = final runoff rate, inches per hour

R = hydraulic radius

S = the energy gradient

t = time

t_o = the time between the beginning of rainfall and the beginning of accumulation of detained water on the land surface

t_c = time of concentration

T = time

$\overline{U} = V$ = mean velocity

$\overline{U}* = (gRS)^{1/2}$ = average boundary shear velocity

$\overline{U}/\overline{U}*$ = Keulegan's mean flow ratio

v = velocity of flow at distance d from the bed

V = average velocity

ν (nu) = kinematic viscosity of the liquid = coefficient of viscosity divided by liquid density